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RESEARCH MEMORANDUM

A CORRELATION OF EXPERIMENTAL ZERO-LIFT DRAG OF RECTANGULAR WINGS WITH SYMMETRICAL NACA 65-SERIES AIRFOIL SECTIONS BY MEANS OF THE TRANSONIC SIMILARITY LAW FOR WINGS OF FINITE ASPECT RATIO

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NATIONAL ADVISORY COMMITTEE
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A CORRELATION OF EXPERIMENTAL ZERO-LIFT DRAG OF RECTANGULAR WINGS WITH SYMMETRICAL NACA 65-SERIES AIRFOIL SECTIONS BY MEANS OF THE TRANSONIC SIMILARITY LAW FOR WINGS OF FINITE ASPECT RATIO

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SUMMARY

A transonic similarity law has been derived in NACA TN 2273, which is applicable to wings of finite aspect ratio and has the interesting feature that the form of the law coincides with one form of the similarity law of linearized theory. One part of this law has been applied to the correlation of the zero-lift drag of rectangular wings obtained by means of the free-fall and rocket techniques. The wings had symmetrical NACA 65-series airfoil sections, exposed aspect ratios between 1.12 and 6.25, and thickness ratios between 0.03 and 0.12.

A slight modification to the similarity law was made to make its application to the correlation of experimental data more convenient. The similarity law in the modified form was found to correlate the data so as to make possible the estimation of the zero-lift drag of rectangular wings other than those tested.

It was also shown, in the interest of clarity, how the principles of similarity may be adapted to swept wings by the inclusion of a sweep parameter identical to that of the linearized theory. The correlation of data for swept wings will in general be more difficult than for straight wings because of the added parameter. The delta wing, however, is a special class of swept wing for which correlation may be affected as easily as for the case of the straight wing.

INTRODUCTION

A similarity law which relates the transonic flow about wings of finite span has been derived in reference 1. One part of this law states that, for all thin wings of a given thickness distribution, and spanwise distribution of chord operating at zero lift and at Mach numbers very near 1.0, the expression for the pressure-drag coefficient may be written as

$$\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}} = f\left[\frac{\sqrt{M^2 - 1}}{\left(\frac{t}{c}\right)^{1/3}},\ A\sqrt{M^2 - 1}\right]$$

For two-dimensional flow the parameter involving aspect ratio may be dropped so that the expression for the pressure-drag coefficient becomes

$$\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}} = g\left[\frac{\sqrt{M^2 - 1}}{\left(\frac{t}{c}\right)^{1/3}}\right]$$

This expression may be shown to be equivalent to the two-dimensional law for transonic flow derived in references 2 and 3. Zero-lift drag data for wings of fairly high aspect ratio have been correlated by means of the two-dimensional law in reference 4.

The results of reference 1, aside from the inclusion of the aspect-ratio parameter, are interesting in another respect. The parameters on which transonic similarity is found to depend are identically those parameters which appear in the similarity laws of linearized theory. Therefore, as was pointed out in reference 1, it should be possible to correlate experimental data, to the first order, by a single principle throughout the subsonic, transonic, and low supersonic regimes without fear that the principle will become inapplicable at any Mach number within these limits.

Sufficient data with which to assess the applicability of the similarity law of reference 1, insofar as the correlation of zero-lift-drag data is concerned, may be found in the results of tests of wings by the free-fall and rocket techniques reported in references 5 to 9. The drag data for rectangular wings with symmetrical NACA 65-series airfoil sections available in these sources have been collected and are correlated herein.

SYMBOLS

A	aspect ratio
A_{ex}	exposed aspect ratio, defined as ratio of square of total exposed span to exposed wing plan area
b	wing span
c	mean geometric chord parallel to plane of symmetry
c_l	local chord parallel to plane of symmetry
C_D	total drag coefficient based on exposed plan area
C_{D_f}	friction-drag coefficient based on exposed plan area
C_{D_p}	pressure-drag coefficient based on exposed plan area, $\left(C_D - C_{D_f}\right)$
f, g, h	functions
M	free-stream Mach number
t	maximum thickness of wing at mean geometric chord
t/c	wing thickness ratio at mean geometric chord
x	dimension in chord plane parallel to plane of symmetry
y	dimension in chord plane perpendicular to plane of symmetry
z	dimension perpendicular to chord plane
Λ	angle of sweep

RESULTS AND DISCUSSION

Basic data.- The basic data to be correlated in the subject paper are presented in figure 1. The total drag coefficient based on exposed wing area is shown as a function of free-stream Mach number for a series of rectangular wings of various aspect ratios and with symmetrical

NACA 65-series airfoils of various thickness ratios. The data for the wings with exposed aspect ratios of 6.25 and 3.75 were obtained by the free-fall technique and were taken from references 5 to 7, while the data for the exposed aspect ratio of 1.12 were obtained by the rocket technique and were taken from references 8 and 9. The rather arbitrary reason for the choice of exposed aspect ratio rather than the total aspect ratio appears later in the discussion.

Several factors influenced the selection of this particular group of data. In the first place, the transonic similarity law is applicable only to wings with affinely related plan forms. Adherence to the requirement that all wings must have the same thickness distribution limits the correlation to wings having airfoil sections of the same family. Transonic test data fulfilling these requirements were most numerous for rectangular wings with symmetrical NACA 65-series airfoil sections. In order that the effects of wing-body interference might be kept to a minimum, only those wing-body data obtained in conjunction with cylindrical bodies have been considered in the analysis.

The data of figure 1 are not ideal from the standpoint of comparison with the theoretical similarity laws. The derivation of these laws does not take into account that the wings have stagnation points, boundary layers, certainly some separation at high subsonic speeds, and at the higher transonic speeds have local velocities that are well above sonic speed. The 65-series-airfoil family is itself not ideal since the leading edge is blunt and the thickness distributions vary slightly with thickness ratio.

The actual flow about wings will always violate to some extent the assumptions involved in the formulation of the similarity laws. The similarity laws themselves are approximations; true similarity is never realized except under identical conditions. In a test of the similarity laws one should not inquire which airfoil shape will best fit the assumptions, but, rather, one should determine to what extent similarity can be realized in practice and to what extent the similarity laws afford a means for the correlation of experimental data. For this broader purpose the use of practical airfoils such as the 65-series is desirable.

Modified similarity rule.- In reference 1 it is shown that for transonic flow over thin wings of a given thickness distribution and spanwise distribution of chord the expression for the zero-lift pressure-drag coefficient may be written as

$$\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}} = f\left[\frac{\sqrt{M^2 - 1}}{\left(\frac{t}{c}\right)^{1/3}},\ A\sqrt{M^2 - 1}\right]$$

In this form the similarity law is not particularly convenient for use in the correlation of experimental data. The difficulty lies in the parameter $A\sqrt{M^2 - 1}$. At a Mach number of 1.0 this parameter becomes zero independently of the aspect ratio. It thus becomes impossible to correlate data at a Mach number of 1.0, since, experimentally, it is found that a strong effect of aspect ratio is there present. The difficulty at $M = 1$ can be avoided, however, by modifying the similarity parameters slightly.

A comparison of equations (18) and (19) of reference 1 will show that, provided the Mach number is different from 1.0, the parameter $A\sqrt{M^2 - 1}$ may be replaced by $A\left(\frac{t}{c}\right)^{1/3}$. For Mach numbers different from 1.0, the similarity law may then be written in modified form as

$$\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}} = g\left[\frac{\sqrt{M^2 - 1}}{\left(\frac{t}{c}\right)^{1/3}}, A\left(\frac{t}{c}\right)^{1/3}\right] \qquad (M \neq 1)$$

which may be shown to be identical in form with the similarity law of linearized theory. Thus, for Mach numbers different from 1.0, at constant values of $\frac{\sqrt{M^2 - 1}}{\left(\frac{t}{c}\right)^{1/3}}$ the drag parameter is a function only of $A\left(\frac{t}{c}\right)^{1/3}$. It might be anticipated that the drag parameter will also depend only upon $A\left(\frac{t}{c}\right)^{1/3}$ at $M = 1$. An unpublished theoretical study by Mr. Harder of the Langley Laboratory does, indeed, show that at $M = 1.0$ the similarity law reduces to

$$\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}} = h\left[A\left(\frac{t}{c}\right)^{1/3}\right] \qquad (M = 1)$$

With full generality, then, the expression

$$\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}} = g\left[\frac{\sqrt{M^2 - 1}}{\left(\frac{t}{c}\right)^{1/3}}, A\left(\frac{t}{c}\right)^{1/3}\right]$$

may be expected to hold at any transonic Mach number.

There is some question of the proper definition of aspect ratio for the case of wings mounted on bodies. It is assumed herein that the body is effective as an end plate and the aspect ratio of the exposed wing A_{ex} is used. Furthermore, the absolute value of $M^2 - 1$ has been chosen under the radical to avoid the appearance of imaginary quantities at Mach numbers less than 1.0.

Correlation of data.- From the data of figure 1, the parameters $\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}}$ and $\frac{\sqrt{|M^2 - 1|}}{\left(\frac{t}{c}\right)^{1/3}}$ have been calculated at many points throughout the test Mach number range for each wing and are plotted in figure 2 as $\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}}$ as a function of $\frac{\sqrt{|M^2 - 1|}}{\left(\frac{t}{c}\right)^{1/3}}$. The data in figure 2 have been plotted as points rather than as faired curves for clarity. In the calculation of the pressure-drag coefficient C_{D_P} an assumed skin-friction drag coefficient 0.006 has been subtracted from the total-drag coefficient in figure 1.

The data in figure 2 are already in the form suggested by the modified transonic similarity law inasmuch as to each wing there corresponds a value of $A_{ex}\left(\frac{t}{c}\right)^{1/3}$. However, it remains to be shown that the variation of $\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}}$ with $A_{ex}\left(\frac{t}{c}\right)^{1/3}$ at particular values of $\frac{\sqrt{|M^2 - 1|}}{\left(\frac{t}{c}\right)^{1/3}}$ is consistent and to interpolate curves representing various values of $A_{ex}\left(\frac{t}{c}\right)^{1/3}$ for use in estimating the drag coefficient of other wings.

The data of figure 2 have been cross-plotted at several values of $\frac{\sqrt{|M^2 - 1|}}{\left(\frac{t}{c}\right)^{1/3}}$ as $\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/2}}$ as a function of $\frac{1}{A_{ex}\left(\frac{t}{c}\right)^{1/3}}$. Samples of these cross plots are shown in figure 3, which also indicates the type of fairing employed. It has been attempted to extrapolate the data to infinite aspect ratio by a linear extrapolation of the fairing to $\frac{1}{A_{ex}\left(\frac{t}{c}\right)^{1/3}} = 0$ as indicated. From plots such as those of figure 3, figure 4 has been prepared in which the drag parameter is shown as a function of $\frac{\sqrt{|M^2 - 1|}}{\left(\frac{t}{c}\right)^{1/3}}$ for various values of $A_{ex}\left(\frac{t}{c}\right)^{1/3}$.

Figure 4 represents the final correlation of the original data of figure 1 according to a modification of the similarity law of reference 1. The correlation is felt to be in a particularly convenient form since the curves of constant $A_{ex}\left(\frac{t}{c}\right)^{1/3}$ each represent the variation of the drag parameter for a possible wing. At any constant value of $\frac{\sqrt{|M^2 - 1|}}{\left(\frac{t}{c}\right)^{1/3}}$ the drag parameter depends only upon the particular combination of aspect ratio and thickness ratio represented by the parameter $A_{ex}\left(\frac{t}{c}\right)^{1/3}$. High values of $A_{ex}\left(\frac{t}{c}\right)^{1/3}$ correspond to high values of the drag parameter. It must be remembered, however, that the assumptions involved in the formulation of the similarity law require that high values of $A_{ex}\left(\frac{t}{c}\right)^{1/3}$ be associated with high values of the aspect ratio only. The thickness ratio must always be small.

It is the sense of the similarity law that figure 4 can be used to estimate the drag of thin rectangular wings of any particular aspect ratio and thickness ratio provided the airfoil section is represented closely by the symmetrical NACA 65-series. In making use of figure 4 it should be kept in mind that the data upon which the correlation was founded extended to values of $A_{ex}\left(\frac{t}{c}\right)^{1/3}$ only as great as about 3.1. The curves in figure 4 for values of $A_{ex}\left(\frac{t}{c}\right)^{1/3}$ of 5 and greater (shown by dashed lines) are extrapolations and as such are uncertain. In order that the use of figure 4 may be simplified, curves from which $\left(\frac{t}{c}\right)^{1/3}$, $\left(\frac{t}{c}\right)^{5/3}$, and $\sqrt{\left|M^2 - 1\right|}$ may be obtained are presented in figure 5.

In order to test the consistency of the fairing employed in the preparation of figure 4, the variation of C_D with M for each of the wings used in the correlation has been recomputed from figure 4 and compared with the original data in figure 6. For the most part, the recomputed data, shown as symbols, agree well with the original data. Since the correlation curves represent the faired composite of data for all the wings, it is not to be expected that local irregularities in the drag curve of any particular wing would be reproduced.

Similarity law for swept wings.- The similarity laws given in reference 1 apply to affine wings whether straight or swept. However, the form of the expression for the thickness distribution used in reference 1 makes the application to swept wings somewhat obscure. In the interest of clarity, it will be shown how the principles of similarity may be adapted to swept wings by the inclusion of a sweep parameter in addition to the aspect-ratio parameter.

Similarity of flow can exist only for those wings with identical nondimensional thickness distributions. In describing the thickness distributions of swept wings in Cartesian coordinates, it is convenient to choose some particular constant chord line as a reference (fig. 7). The sweep of the wing may then be defined as the sweep of this constant chord line and distances in the chord plane parallel to the plane of symmetry may be measured with respect to this line. The thickness distribution of a swept wing may thus be written as

$$\frac{z}{t} = f\left(\frac{x - y \tan \Lambda}{c_l}, \frac{y}{b}\right)$$

The identity of thickness distribution for a series of swept wings thus requires the identity of the boundaries of the nondimensional plan forms defined by

$$f\left(\frac{x - y \tan\Lambda}{c_l}, \frac{y}{b}\right) = 0$$

or

$$f\left[\left(\frac{x}{c_l} - \frac{c}{c_l}\frac{y}{b}\frac{b}{c}\tan\Lambda\right), \frac{y}{b}\right] = 0$$

or since $\frac{b}{c} = A$,

$$f\left[\left(\frac{x}{c_l} - \frac{c_l}{c}\frac{y}{b} A \tan\Lambda\right), \frac{y}{b}\right] = 0$$

Thus, flow similarity for swept wings requires that the airfoil sections in the plane of symmetry be of the same family and that the spanwise distribution of chord be identical just as in the case of straight wings. In addition, when sweep is introduced, it is required that the product $A \tan\Lambda$ be equal if flow similarity is to exist. The angle of sweep must always be measured with respect to the same constant chord line. Since it has already been shown that flow similarity requires equality of the parameter $A\left(\frac{t}{c}\right)^{1/3}$, it is evident that the parameter $\frac{\tan\Lambda}{\left(\frac{t}{c}\right)^{1/3}}$ must also be equal for swept wings.

The transonic similarity law for the zero-lift drag of swept wings may now be written as

$$\frac{C_D}{\left(\frac{t}{c}\right)^{5/3}} = g\left[\frac{\sqrt{M^2 - 1}}{\left(\frac{t}{c}\right)^{1/3}}, A\left(\frac{t}{c}\right)^{1/3}, \frac{\tan\Lambda}{\left(\frac{t}{c}\right)^{1/3}}\right]$$

which may be shown to coincide with one form of the similarity law of linearized theory. At a Mach number of 1.0 this expression reduces to

$$\frac{C_D}{\left(\frac{t}{c}\right)^{5/3}} = h\left[A\left(\frac{t}{c}\right)^{1/3}, \frac{\tan \Lambda}{\left(\frac{t}{c}\right)^{1/3}}\right]$$

Parallel relations may be written for the lift and moment coefficients.

The correlation of experimental data for swept wings will in general be more difficult than for the case of straight wings because of the additional similarity parameter involved. In the case of the delta wing, however, the aspect ratio is inversely proportional to the tangent of the angle of sweep so that either the aspect-ratio parameter or the sweep parameter may be dropped in preference to the other. The correlation of experimental data for delta wings, then, should be no more difficult than for the case of straight wings.

CONCLUDING REMARKS

One part of the transonic similarity law of NACA TN 2273 has been applied to the correlation of zero-lift drag data of rectangular wings obtained in conjunction with cylindrical bodies by means of the free-fall and rocket techniques. The wings had symmetrical NACA 65-series airfoil sections, exposed aspect ratios between 1.12 and 6.25, and thickness ratios between 0.03 and 0.12.

A slight modification to the similarity law of NACA TN 2273 was made to avoid an indeterminacy at a Mach number of 1.0. The similarity law in the modified form was found to correlate the data so as to make possible the estimation of zero-lift drag of rectangular wings other than those tested.

It was also shown, in the interest of clarity, how the principles of similarity may be adapted to swept wings by the inclusion of a sweep parameter identical to that of the linearized theory. The correlation for swept wings will, in general, be more difficult than for straight wings because of the additional parameter. The delta wing, however, is a special class of swept wing for which correlation may be effected as easily as for the case of the straight wing.

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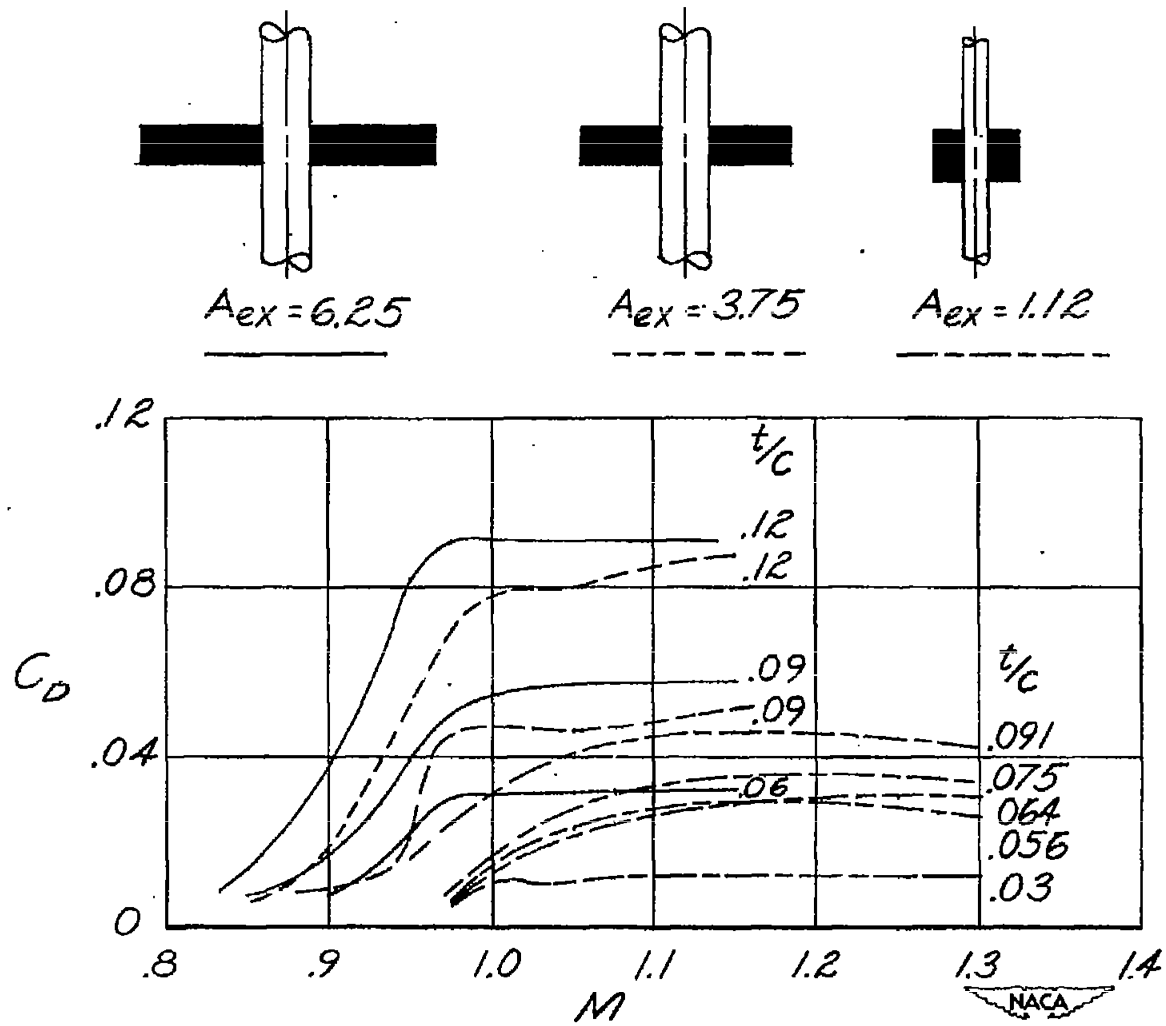


Figure 1.- Variation of zero-lift drag coefficient with Mach number for several straight wings of different aspect ratio with NACA 65-0XX series airfoil sections.

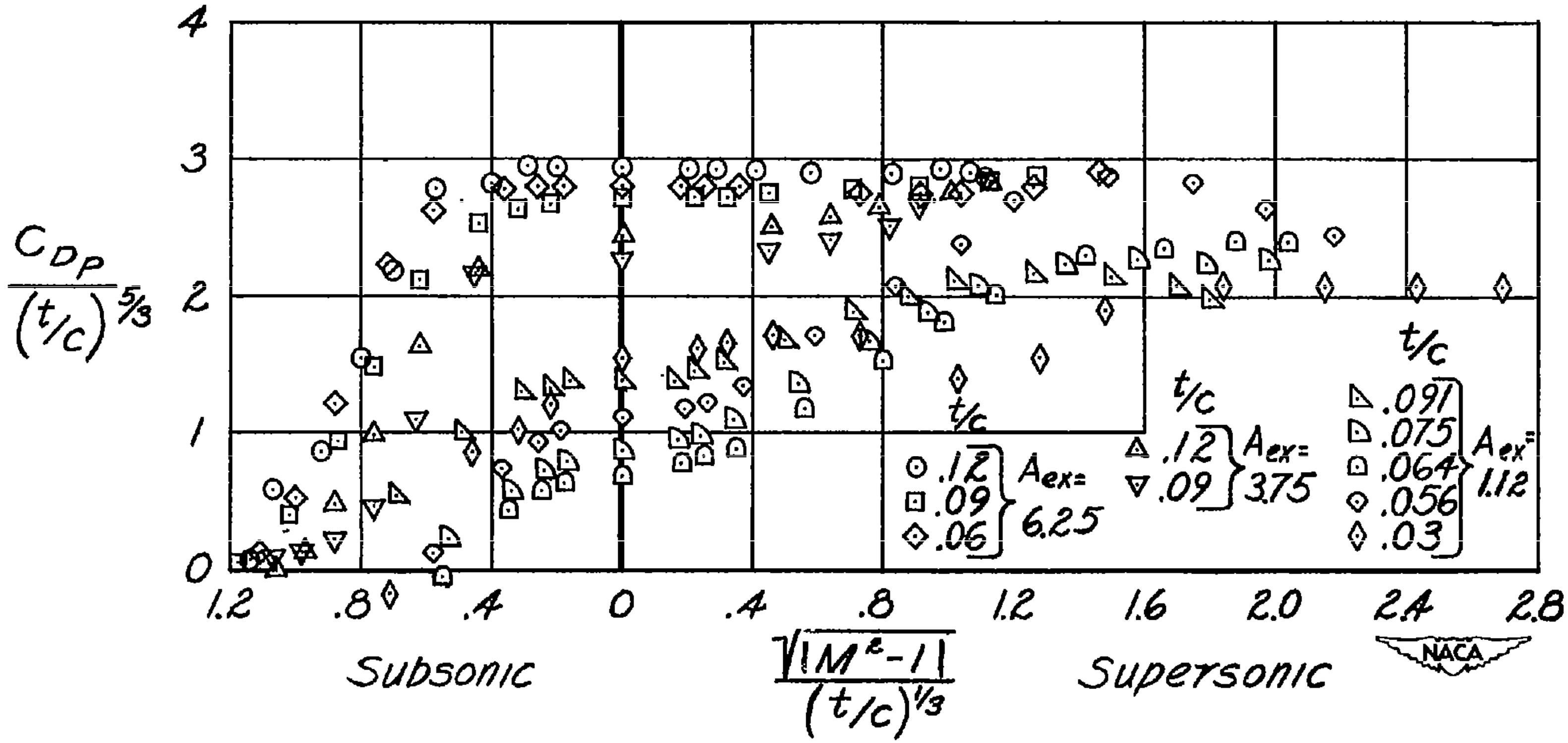


Figure 2.- Variation of $\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}}$ with $\frac{\sqrt{|M^2 - 1|}}{\left(\frac{t}{c}\right)^{1/3}}$

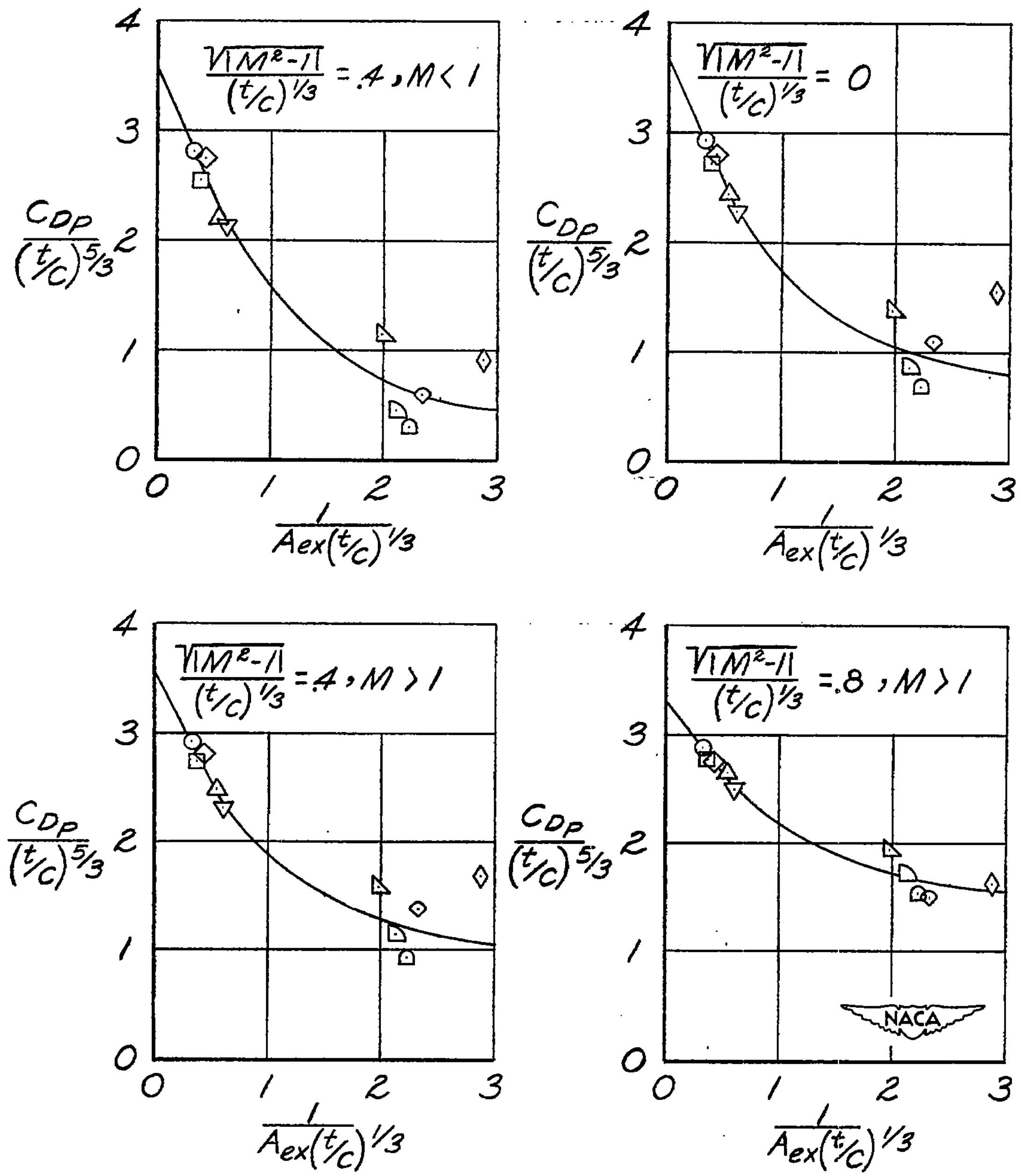


Figure 3.- Variation of $\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}}$ with $\frac{1}{A_{ex}\left(\frac{t}{c}\right)^{1/3}}$ at several values of $\frac{\sqrt{|M^2 - 1|}}{\left(\frac{t}{c}\right)^{1/3}}$. The symbols are those defined in figure 2.

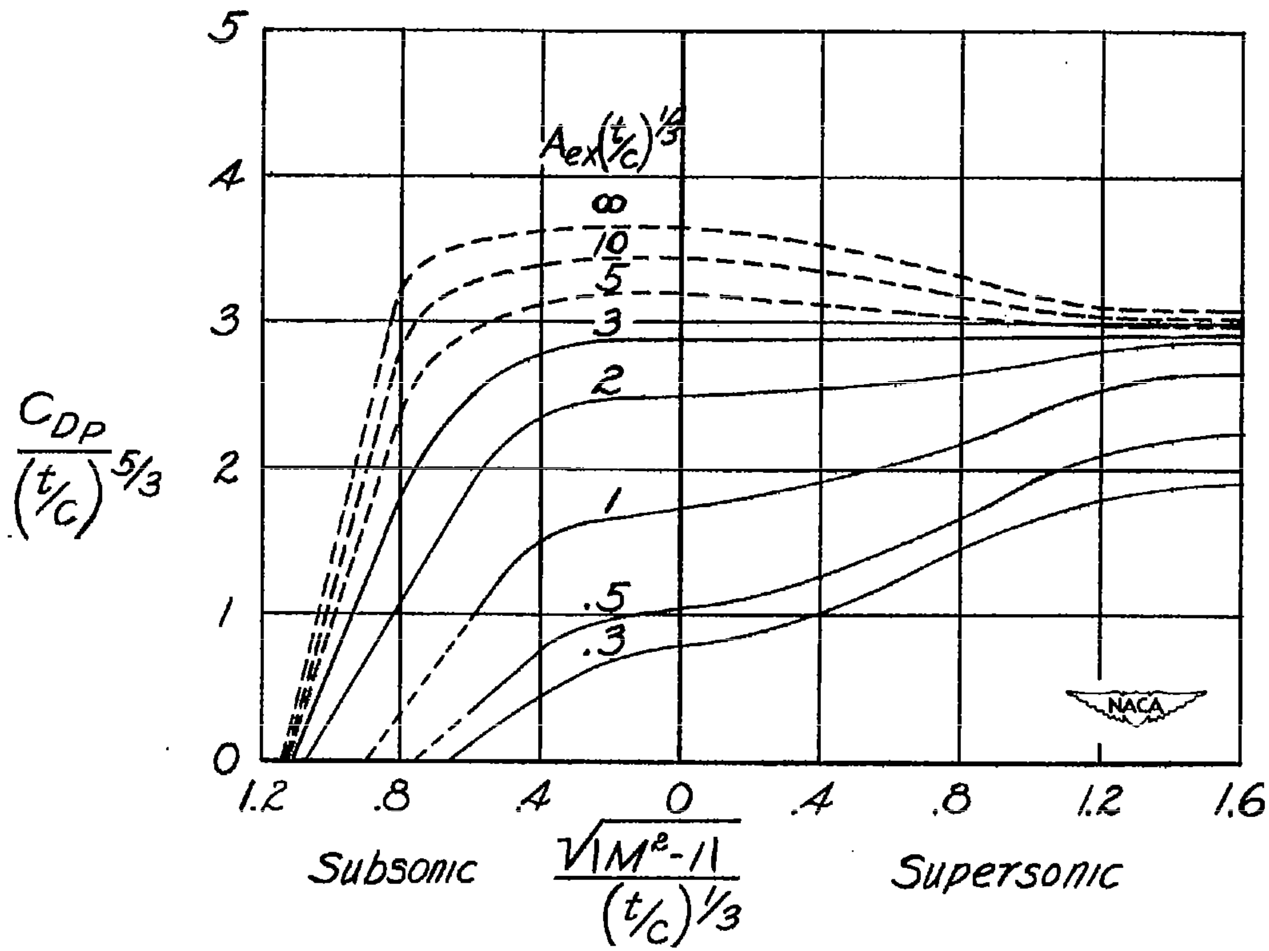
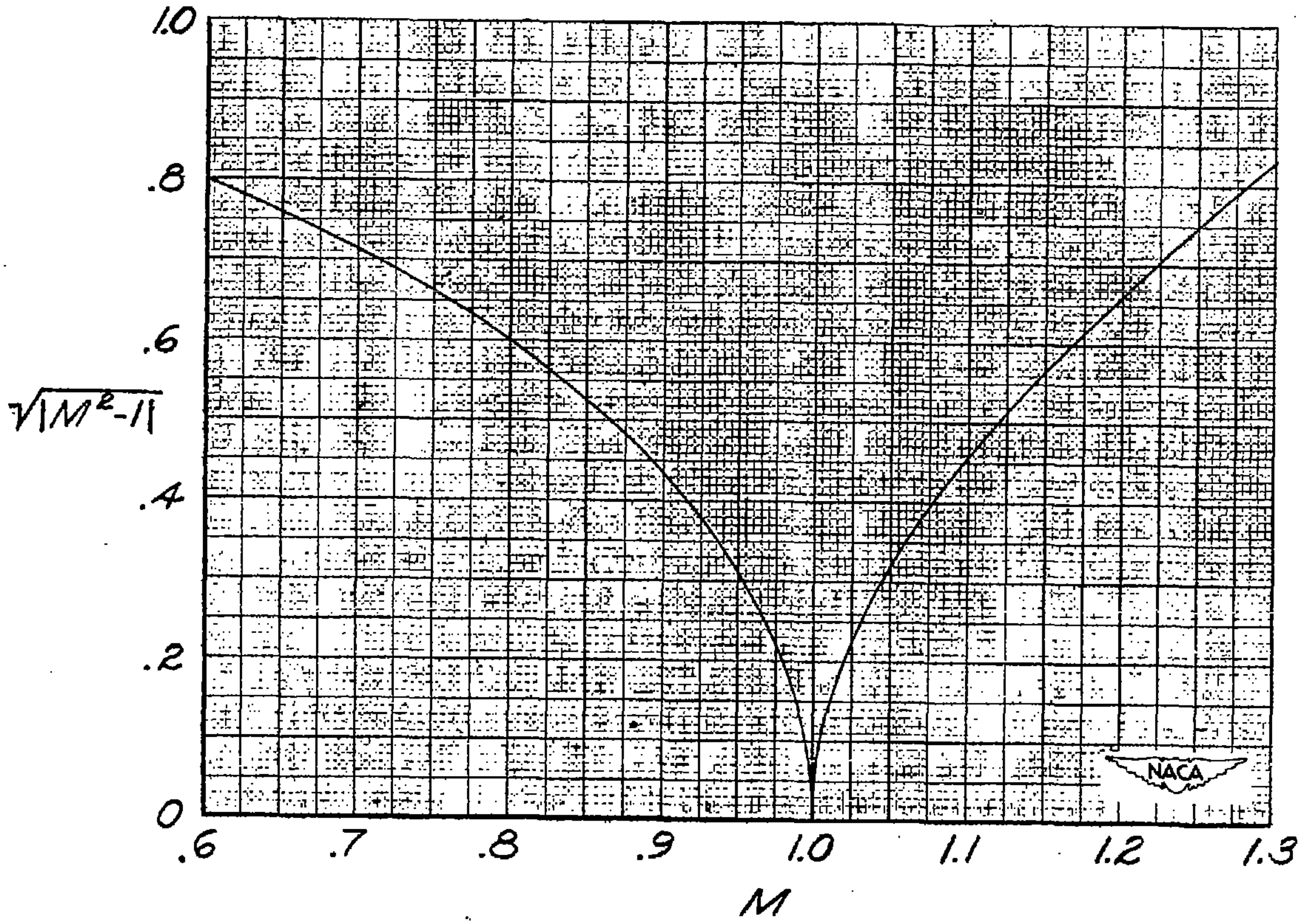
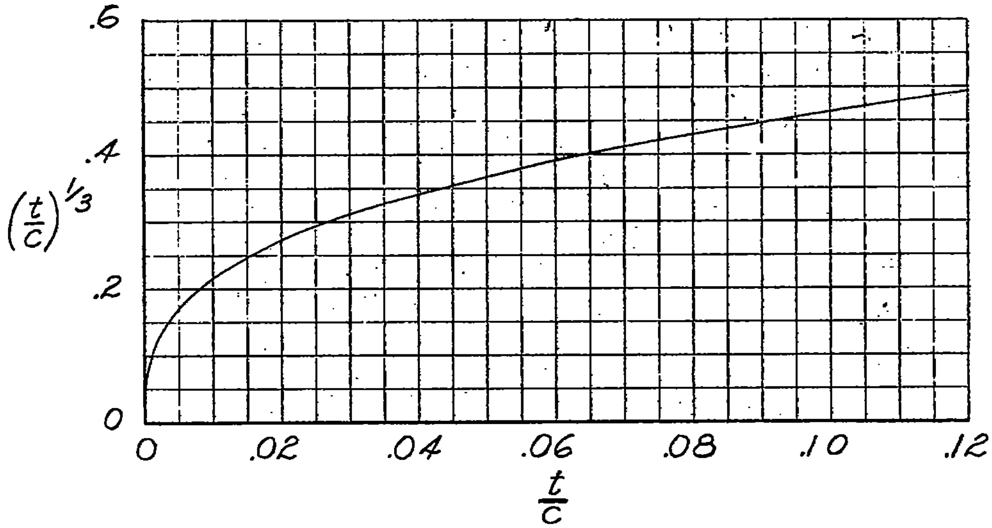


Figure 4.- Variation of $\frac{C_{D_P}}{\left(\frac{t}{c}\right)^{5/3}}$ with $\frac{\sqrt{|M^2 - 1|}}{\left(\frac{t}{c}\right)^{1/3}}$ and $A_{ex}\left(\frac{t}{c}\right)^{1/3}$ for NACA 65-0XX airfoils.

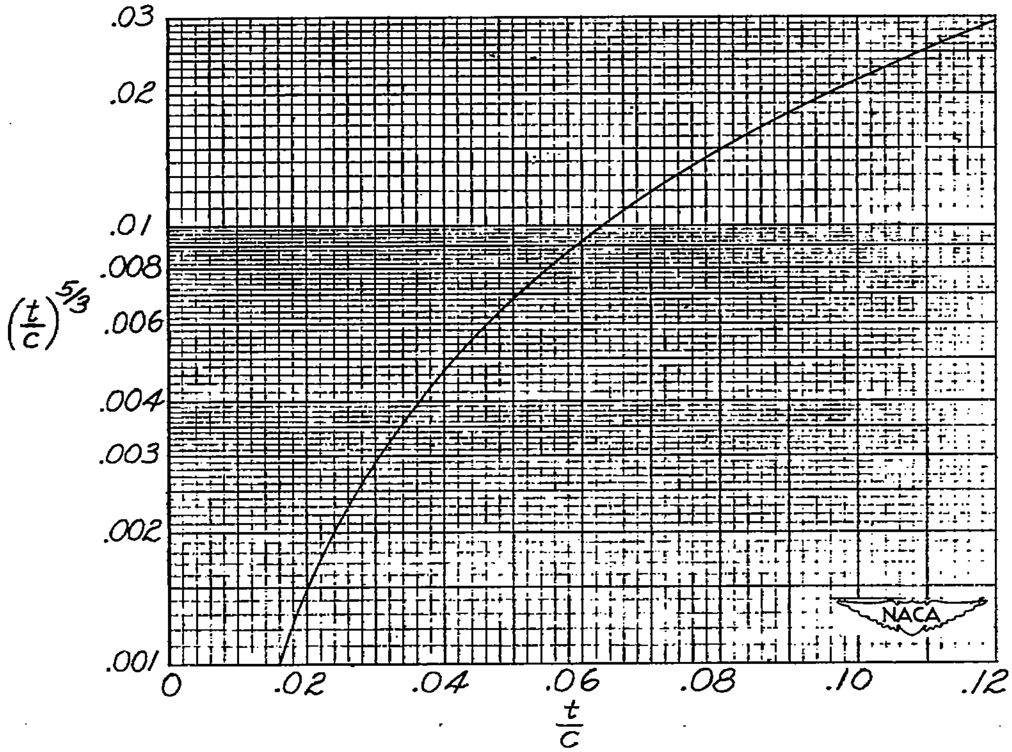


(a) Variation of $\sqrt{|M^2 - 1|}$ with M.

Figure 5.- Functions for use in connection with figure 4.

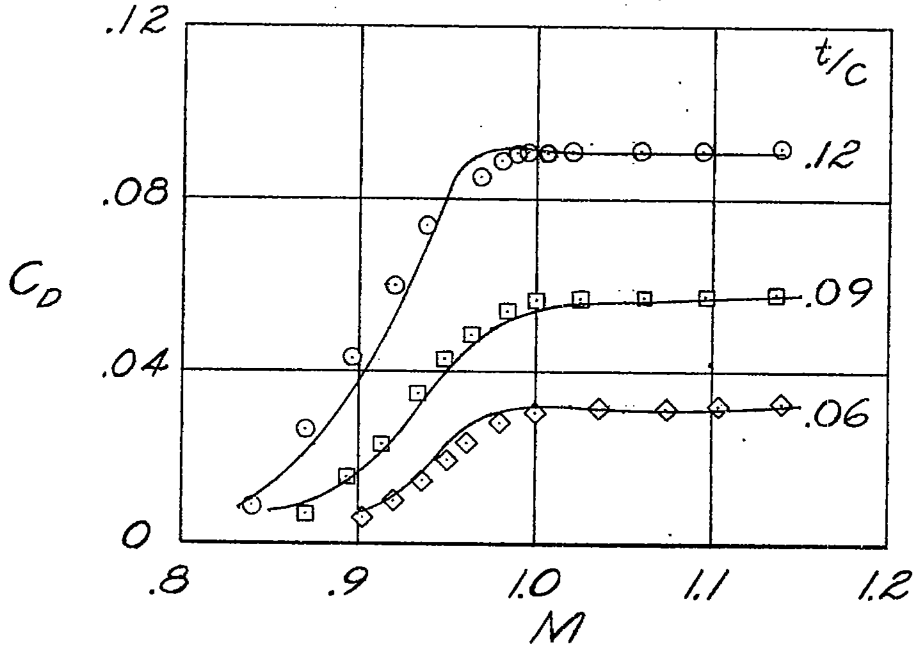


(b) Variation of $\left(\frac{t}{c}\right)^{1/3}$ with $\left(\frac{t}{c}\right)$.

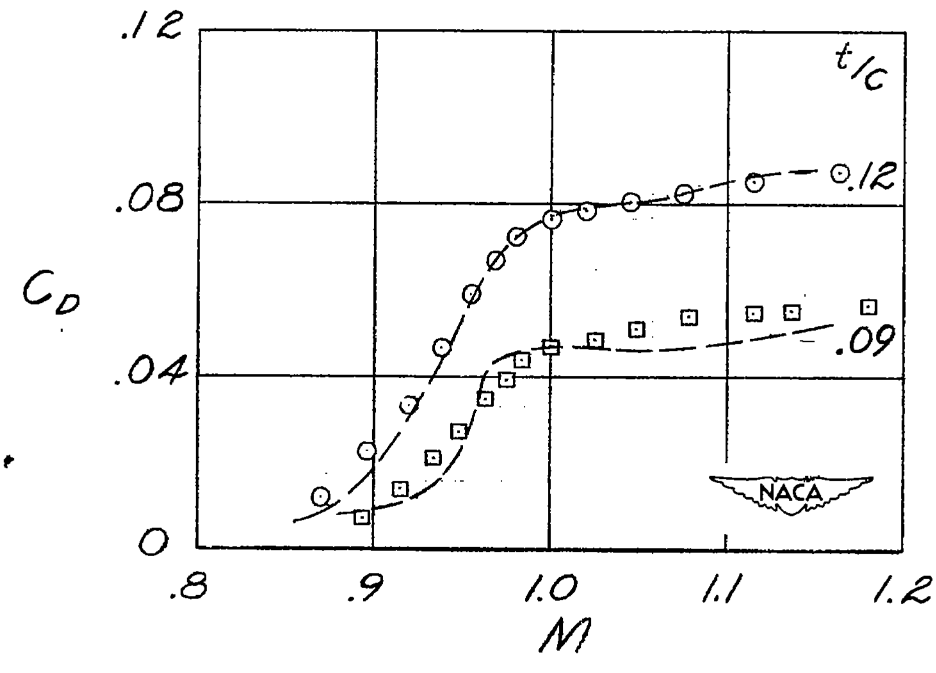


(c) Variation of $\left(\frac{t}{c}\right)^{5/3}$ with $\left(\frac{t}{c}\right)$.

Figure 5.- Concluded.

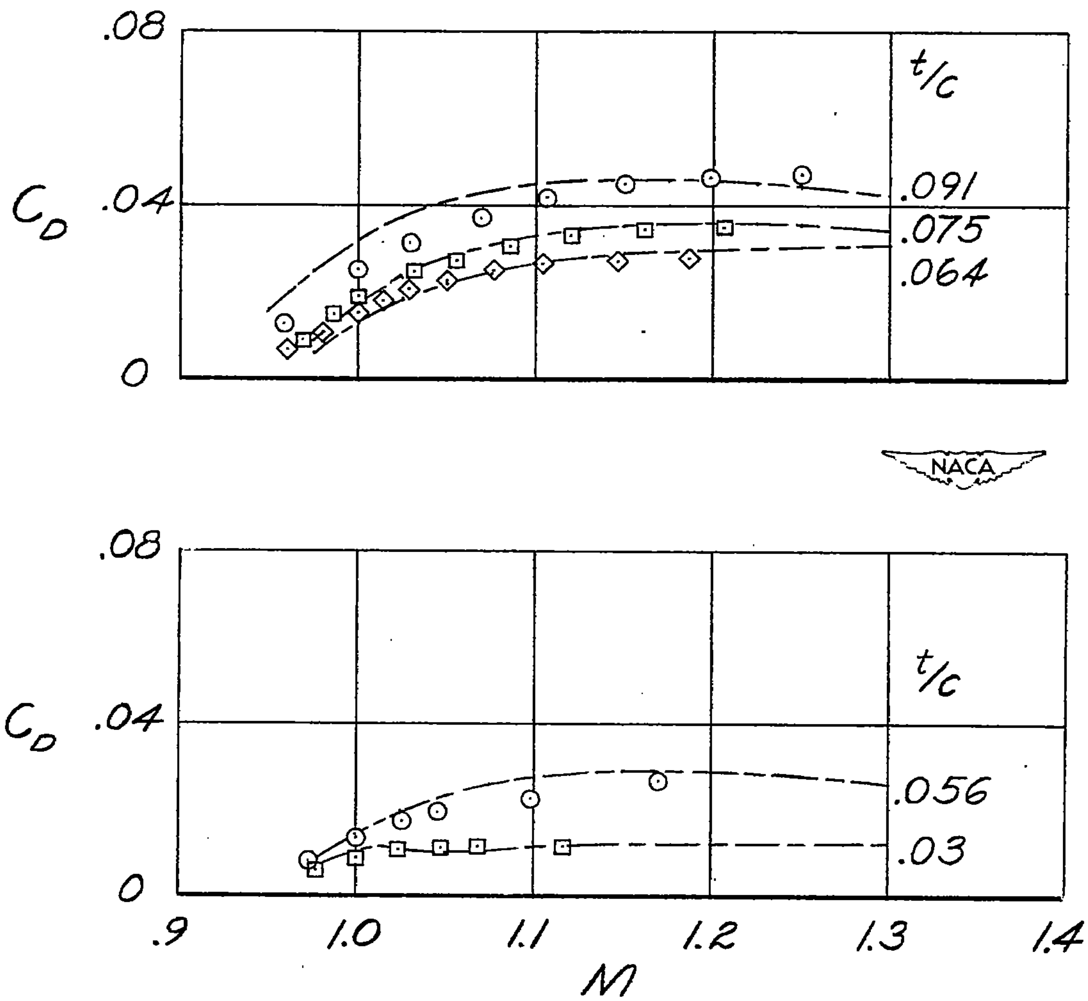


(a) $A_{ex} = 6.25$.



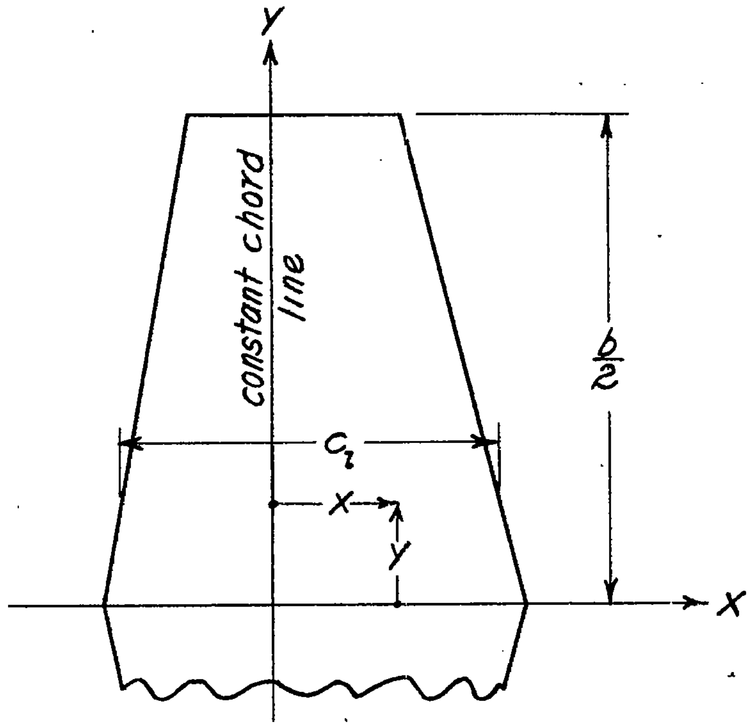
(b) $A_{ex} = 3.75$.

Figure 6.- Comparison of the variation of C_D with M computed from figure 4 with the original data of figure 1.

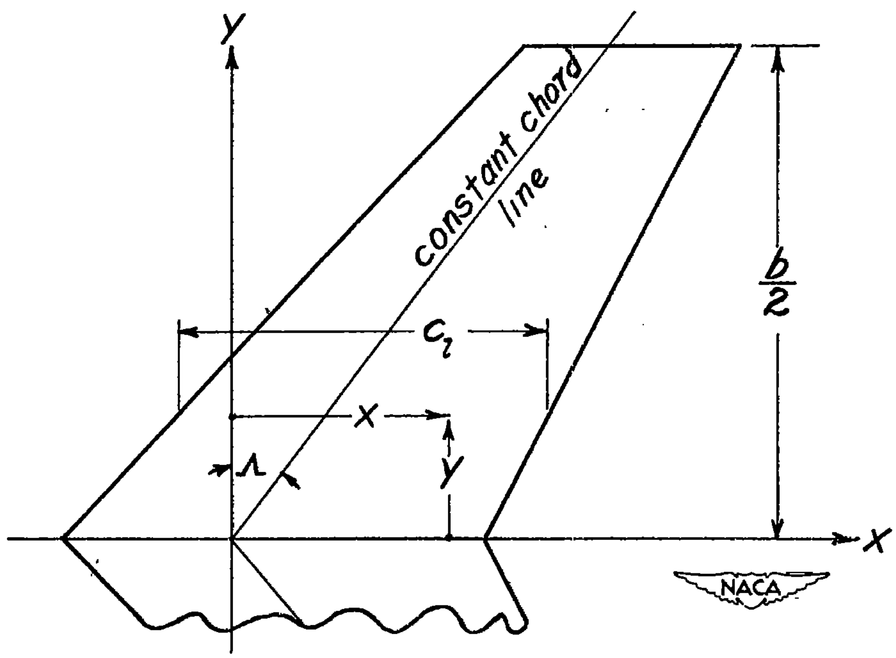


(c) $A_{ex} = 1.12$.

Figure 6.- Concluded.



(a) Straight wings.



(b) Swept wings.

Figure 7.- Coordinate systems used in defining thickness distribution.

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